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AN ENTIRE FUNCTION SHARING A POLYNOMIAL WITH LINEAR DIFFERENTIAL POLYNOMIALS

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Abstract. The uniqueness problems on entire functions sharing at least two values with their derivatives or linear differential polynomials have been studied and many results on this topic have been obtained. In this paper, we study an entire function $f(z)$ that shares a nonzero polynomial $a(z)$ with $f^{(1)}(z)$, together with its linear differential polynomials of the form: $L = L(f) = a_1(z)f^{(1)}(z) + a_2(z)f^{(2)}(z) + \cdots + a_n(z)f^{(n)}(z)$, where the coefficients $a_k(z)(k = 1, 2, \ldots, n)$ are rational functions and $a_n(z) \not\equiv 0$.

1. Introduction, definitions and results

In the paper, by meromorphic functions we shall always mean meromorphic functions in the complex plane $\mathbb{C}$. We adopt the standard notations of the Nevanlinna theory of meromorphic functions as explained in [2]. It will be convenient to let E denote any set of positive real numbers of finite linear measure, not necessarily the same at each occurrence. For a non-constant meromorphic function h, we denote by $T(r, h)$ any quantity satisfying $S(r, h) = o\{T(r, h)\}$ as $r \to \infty$ and $r \notin E$.

Let f and g be two nonconstant meromorphic functions and let a be a small function of f. We denote by $E(a; f)$ the set of a-points of f, where each point is counted according its multiplicity. We denote by $\overline{E}(a; f)$ the reduced form of $E(a; f)$. We say that f, g share a CM, provided that $E(a; f) = E(a; g)$, and we say that f and g share a IM, provided that $\overline{E}(a; f) = \overline{E}(a; g)$. In addition, we say that f and g share ∞ CM, if $\frac{1}{f}$ and $\frac{1}{g}$ share 0 CM, and we say that f and g share ∞ IM, if $\frac{1}{f}$ and $\frac{1}{g}$ share 0 IM.

In 1977, L. A. Rubel and C. C. Yang [8] first investigated the uniqueness of entire functions, which share certain values with their derivatives. The following is the result of Rubel and Yang [8].

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